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Mixing in Stratified Fluids†

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Laboratory experiments on mixing in stratified fluids are examined in the light of some simple energy arguments applied to an analysis of mixing at density interfaces. It is shown that as the stability of the flow, as measured by an overall Richardson number Ri_0, increases, the velocity interface becomes considerably thicker than the density interface. This mismatch in interface thickness provides the extra kinetic energy required for mixing in these strongly stratified flows. The behaviour of the flux Richardson number Rf as a function of Ri_0 is also discussed. It is found that the results from a number of different experiments are quite similar, and that Rf increases from zero as Ri_0 does, reaches a maximum value (0.2 ± 0.05) and then decreases again with further increase in Ri_0. Finally, some recent ideas of Posmentier (1977) on the formation of interfaces in a stratified, turbulent flow are compared with observations.

1. INTRODUCTION

The fact that stable density stratification can have a substantial effect on mixing will be obvious to most of us. It is only necessary to visit a large industrial or urban complex when there is an atmospheric inversion to see its influence. The stable stratification in the inversion layer inhibits the vertical mixing of pollutants keeping them concentrated near the ground, often producing unpleasant conditions. Indeed, a large part of the atmosphere is stably stratified and so the way in which mixing takes place in such a environment is an important determining feature of the global distributions of heat and water vapour. The oceans, too, are stably stratified with the warmest water at the top, and many phenomena depend upon the rate of vertical transports of heat and salt due to turbulent mixing. A large number of industrial processes also involve mixing in stably stratified environments. In the home stable density stratification is a

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common feature with the warmest air near the ceiling of a room. The way in which the air mixes in such a room is very important for our comfort.

The dynamical effects of the stratification manifest themselves as a buoyancy force which acts as a restoring force whenever fluid is displaced vertically from its equilibrium position. In order to mix fluid vertically it is necessary to do work against this buoyancy force, and this work is extracted from the agency doing the mixing. If the fluid is being mixed by a turbulent flow, more energy may be required to work against the buoyancy force than is available and the character of the flow may be altered dramatically.

In order to estimate the effects of the buoyancy force we need to relate it to the forces available to mix the fluid. This is conveniently done by considering the flow between two horizontal planes separated by a vertical distance D. We suppose that the lower plane is moving horizontally with a velocity ΔU relative to the upper, and that fluid at the lower plane is maintained at a density difference $\Delta\rho$ relative to that at the upper boundary. As we are interested in stable stratification, $\Delta\rho > 0$. If g is the gravitational acceleration and ρ the mean density, the stratification is characterised by the non-dimensional variable $Ri_0 \equiv g\Delta\rho D/\rho(\Delta U)^2$. We will call this an overall Richardson number as it relates to the gross parameters of the system.

Suppose a flow is set up so that the vertical gradients of density and velocity (i.e. $\Delta\rho/D$, $\Delta U/D$) are constant. If we move a fluid parcel a small distance, d, then the change in potential energy is $g\Delta\rho d^2/2D$ and the available kinetic energy is $\rho(\Delta U d)^2/2D^2$. The ratio of potential energy change to kinetic energy change is then Ri_0. Consequently, if $Ri_0 > 1$ more energy is required to mix the fluid in this manner than is available in the velocity chear. This result is independent of the vertical scale of the proposed motion.

But we know, both from laboratory and field measurements, that mixing does take place when Ri_0 is large. In the ocean, for example, $Ri_0 = 0(10)$ or larger, and yet turbulence and mixing have often been observed. We are led in such circumstances to relax the one constraint we have imposed, namely, that the mean gradients of density and velocity are constant. Hence, there must be non-uniformities in the velocity and density profiles. So we see that the presence of layers and interfaces is intimately bound up with turbulence in regions of high Ri_0. Then, Turner (1973, p. 320) has shown, we can match any local gradient Richardson number with any Ri_0 essentially because $\overline{(\partial U/\partial z)^2} \gg (\Delta U/D)^2$.

There are many processes associated with density interfaces which have been identified in recent years, and a large fraction of the reasearch effort in this subject has been (and still is) the study of these individual processes. Some examples are

(i) Kelvin–Helmholtz billows
(ii) Breaking internal and interfacial waves
(iii) Lateral intrusions
(iv) Double-diffusive convection

These, and other, processes have been discussed in a recent review by Sherman, Imberger and Corcos (1978).

In this review I intend to look at the subject from a different viewpoint. One difficulty of treating each process individually is that it does not readily yield any general principles about mixing in stratified fluids. It is often hard to relate one process to another or to fit them into any general framework. This is reflected in the larger number of different experiments that have been carried out in the last twenty years or so. Often seemingly conflicting results are obtained from what appear to be similar situations and at present it is often difficult to obtain answers to apparently simple questions.

In order to arrive at a more unified view some simple energy arguments will be given for mixing at a density interface. In the light of these arguments a number of different experiments will be examined. In so doing attention will be restricted to the case where the driving force for the mixing is mechanical in origin; for example, by driving a shear flow or stirring the fluid in some way. Mixing by convective processes is excluded. The restriction of considering mixing at an interface is made for convenience. It is not a very severe restriction, however, as it recognises the fact that interfaces are an inevitable consequence of mixing at high Ri_0. In this respect we are dealing with mixing and turbulence when the stratification is dominant: the situation is *not* a small perturbation from turbulence in a homogeneous fluid.

This raises another interesting question. In their investigations of the entrainment into a steady gravity current flowing down a slope, Ellison and Turner (1959) found that the entrainment virtually ceased at high values of Ri_0. Thus we have the possibility that the stratification can cause a collapse of the turbulence when it is strong enough. The partition of energy during mixing in strongly stratified flows is examined in section 4. A comparison is made between a number of different mixing processes, and it is found that there is considerable similarity between them. The final part of the paper shows how the behaviour of the flux Richardson number with stability influences the formation of interfaces caused by mixing.

2. ENERGETICS OF INTERFACIAL MIXING

In any mixing event in a stably stratified fluid. work is done to change the potential energy of the fluid. This is recognised by the raising of the centre mass of the fluid column as mixing occurs. It is necessary, therefore, that there be enough energy available to accomplish this mixing. This places quite severe, and revealing, restrictions on the possible outcome of mixing events. We will deal with two cases.

(a) The splitting of an interface

It has been suggested (Woods and Wiley, 1972) that Kelvin–Helmholtz instability on an interface may cause it to mix to form a uniform layer with sharp interfaces on either side. This case has been discussed by Turner, (1973, p. 322) and he shows that if initial linear gradients

$$u = \alpha z, \qquad \rho = \rho_0 - \beta z$$

split into a well-mixed layer of thickness H, the change in potential energy is $-g\beta H^3/12$ and the change in kinetic energy is $\alpha^2 H^3/24$. These two are equal in magnitude when $Ri_0 = g\beta/\alpha^2 = 0.5$. Even ignoring viscous dissipation, an additional supply of kinetic energy is required to perform this mixing when $Ri_0 > 0.5$.

(b) Altering the thickness of an interface

Observations by Thorpe (1973) indicate that mixing produced by Kelvin–Helmholtz instability leads to an approximately linear variation in density and velocity across the interface, but that the interface has changed in thickness. Similar behaviour has been observed in other experiments and this is the case discussed below.

Suppose, as shown on Figure 1, that stratified fluid of total depth 2D consists of two layers with uniform density and velocity separated by an interface in which the gradients of density and velocity are constant. We denote the density and velocity of the lower layer by ρ_1 and U_1, respectively, and those of the upper layer by ρ_2 and U_2. As a result of mixing it will be supposed that the interface changes its thickness symmetrically about the centre line $z = D$, but that the two layers on either side remain uniform.

Conserving mass and horizontal momentum we find

$$\text{Initial K.E.} - \text{Final K.E.} = \tfrac{1}{6}\rho(\Delta U)^2(d_f - d_i); \tag{2.1}$$

$$\text{Initial P.E.} - \text{Final P.E.} = \tfrac{1}{6}g\{\Delta\rho_i\delta_i^2 - \Delta\rho_f\delta_f^2 - 3D^2(\Delta\rho_i - \Delta\rho_f)\}. \tag{2.2}$$

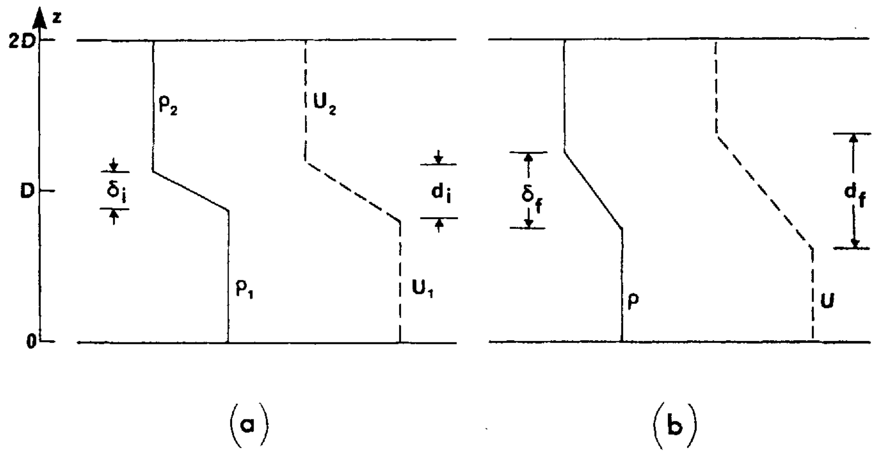


FIGURE 1. A diagrammatic representation of the vertical profiles of density (solid line) and velocity (broken line), before (a) and after (b) a mixing event. It is assumed that after mixing the profiles have the same form, although the thicknesses of the transition regions have been altered.

Here kinetic energy is abbreviated to K.E. and potential energy to P.E.,

$$\Delta\rho = \rho_1 - \rho_2,$$

$$\Delta U = U_1 - U_2,$$

d = thickness of the velocity interface,

δ = thickness of the density interface.

The subscripts i and f refer to the initial and final states, respectively. It has been assumed that ΔU remains unaltered; this is done simply for convenience, and it turns out to be a good approximation in the experiments to be discussed later.

Before examining the experiments in detail we can draw two simple conclusions from (2.1) and (2.2). From (2.1) we see that energy is only released from the velocity field if $d_f > d_i$. Consequently, if an interface is observed to get thinner as a result of mixing there must be some additional source of energy. Indeed we need some turbulence in the layers to redistribute the mass from the interface through the layers.

If, on the other hand, the thickness of the interface merely increases without altering the densities in the layers (as in Kelvin–Helmholtz instability), the fact that the change in potential energy must be less than the available kinetic energy implies

$$\frac{g\Delta\rho(\delta_f^2-\delta_i^2)}{\rho(\Delta U)^2(d_f-d_i)}<1. \tag{2.3}$$

If the velocity and density scales are equal then

$$\frac{g\Delta\rho(d_f+d_i)}{\rho(\Delta U)^2}<1. \tag{2.4}$$

Thus we have a strict limit on the ultimate thickness an interface can reach by such a process.

3. COMPARISON WITH LABORATORY EXPERIMENTS

In this section a number of different laboratory experiments will be discussed in the light of the above energy argument. Only a very brief description of the experiments will be given; for further details the reader is referred to the original papers.

The first class of experiments concerns the mixing at an interface between two horizontal layers of fluid moving with different horizontal velocities. These were performed by Koop (1976) who brought the two layers together at the edge of a splitter plate, and observed the downstream evolution as mixing between the two streams developed. As the Prandtl number ($\sigma\equiv\nu/\kappa$; kinematic viscosity/molecular diffusivity) for his flow was $0(10^3)$, the initial thickness of velocity interface was much larger than the density interface. So to a good approximation $\delta_i=0$, and the flow is characterised by an overall (or initial) Richardson number

$$Ri_I=\frac{g\Delta\rho d_i}{\rho(\Delta U)^2},$$

where ΔU is the velocity difference between the layers.

From (2.3) we see that, with $\delta_i=0$,

$$\left(\frac{d_f}{\delta_f}\right)^2>\frac{Ri_I}{\left(1-\frac{d_i}{d_f}\right)\frac{d_i}{d_f}}.$$

Now $x(1-x)\leqq\frac{1}{4}$ for $0<x<1$ and so

$$\frac{d_f}{\delta_f}>2Ri_I{}^{1/2}. \tag{3.1}$$

Thus, as Ri_I increases, the final velocity interface becomes appreciably thicker than the final density interface. This is a result of the fact that at high stabilities it is necessary to decelerate more of the flow in order to mix the fluids.

It is not possible to do much more than make a qualitative comparison with Koop's results, due to ambiguities in determining d_f/δ_f caused by the presence of waves and molecular diffusion which continue to thicken the interface after the turbulence has decayed. In any case (3.1) represents only a strict lower bound on d_f/δ_f as it ignores viscous dissipation. However, Koop finds that d_f/δ_f increases from approximately 2 at the lowest initial Richardson number to about 7 at the largest, in qualitative agreement with these ideas.

Another experiment in this class was performed by Moore and Lọng (1971). They produced two layer, counter-current flow in an annular tank. They drove the flow by small directed jets of fluid injected at the upper and lower boundaries, and were able to set up steady-state mixing experiments by simultaneously withdrawing fluid at these boundaries. Experiments were run over a range of overall Richardson number $0.7 < Ri_0 < 61.5$, where $Ri_0 = (g\Delta\rho D)/\rho(\Delta U)^2$. Here D is the depth of the channel, ΔU the velocity difference and $\Delta\rho$ the density difference between the layers. Vertical profiles of velocity and density across the channel were measured and a number of interesting results emerge. The first is shown on Figure 2 which is a plot of data contained in Table 1 of their paper. The figure shows the steady-state thickness of the velocity interface as a fraction of the channel depth plotted against Ri_0. At low values of Ri_0, the interface occupies up to 80% of the channel width, but it becomes thinner as Ri_0 increases. For values or $Ri_0 > 10$ the thickness becomes roughly constant with $d/D \approx 0.1$.

Suppose, for the moment, that the steady-state velocity and density interfaces have equal thickness. Then assuming $d_i = 0$ and writing $d = d_f$ in (2.4), we get

$$\frac{g\Delta\rho d}{\rho(\Delta U)^2} < 1.$$

Alternatively,

$$\frac{d}{D} < Ri_0^{-1}. \tag{3.2}$$

The curve $d/D = Ri_0^{-1}$ is shown on Figure 2. The fact that this curve and the data points lie close together has some important consequences. For

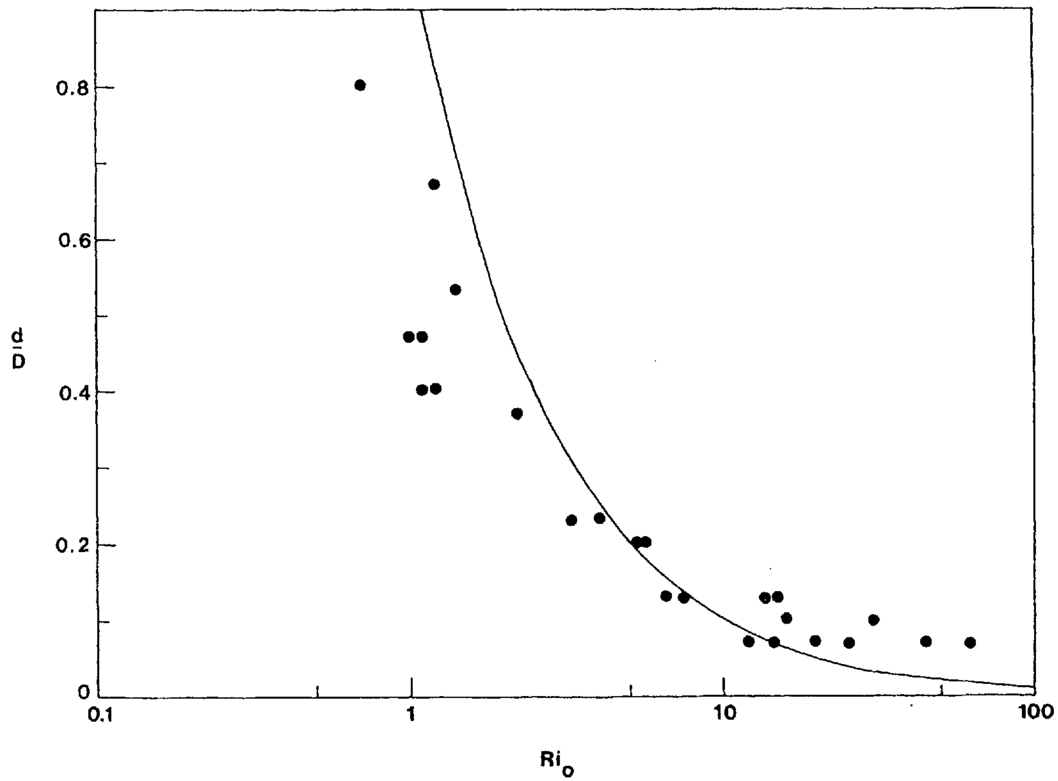


FIGURE 2 The thickness of the velocity interface d, non-dimensionalised with respect to the depth of the channel D, plotted as a function of the overall Richardson number Ri_0. These data were obtained by Moore and Long (1971). The curve is $d/D = Ri_0^{-1}$.

$Ri_0 < 10$, it seems that the upper bound on d/D given by (3.2) is almost attained. This means that the steady-state thickness is limited by the energetics of the mixing which has been derived *ignoring viscous dissipation.* This is presumably because, although energy is continuously being lost to dissipation it is continually supplied by forcing the flow. This is in contrast to mixing experiments in which the energy supply is limited (as in Koop's case), where the viscous dissipation severely reduces the amount of energy for mixing and must be considered.

For $Ri_0 > 10$ we see that d/D violates the criterion given in (3.2) and, as we have already noted, the interface thickness remains roughly constant. Now (3.2) implies that $d/D \to 0$ as $Ri_0 \to \infty$. In an apparatus of finite size this implies that thickness of the velocity interface vanishes, and so the gradient Richardson number must vanish as $Ri_0 \to \infty$. However, then the interface would be unstable to Kelvin–Helmholtz billows, and this leads to a thickening of the interface. This is a contradiction and so it is necessary to violate (3.2). The only way to do this is to relax the supposition that the velocity and density interfaces have the same length scales. This is what is observed. From the profiles published in their paper we see that $d/\delta = 1.5$, 2.0 and 3.0 for $Ri_0 = 2.2$, 4 and 15, respectively. Thus, as in Koop's experiment, at high stabilities the velocity interface becomes significantly thicker than the density interface, which allows the flow to become unstable at all values of Ri_0.

Another feature of Moore and Long's experiment which deserves comment is the fact that they were able to set up and maintain a steady-state. It was noted in section 2 that mixing produced by the mean shear continually increases the thickness of the interface. So it is essential to have some way of removing mixed fluid from the interface in order to keep its thickness constant. In their experiment this is provided by turbulence in the layers produced by the jets of injected fluid at the top and bottom boundaries of the tank. Thus we see that forcing the flow in this way both maintains the shear across the interface and also produces a flux of density across the layers on either side of the interface. This seems to be the reason why the criterion (3.2) applies to this flow. It is envisaged that from an initially sharp interface, once the forcing is begun the thickness of the interface increases until the limit described by (3.2) is reached. Further increase in d is not possible as there is insufficient energy in the mean shear, and the turbulence in the layers, by removing mixed fluid, is tending to reduce d rather than increase it.

A consideration of the small scale turbulence induced by the jets in Moore & Long's experiments leads us to the second class of experiments which can be discussed in these terms. These are flows in which the two layers are stirred mechanically, but where there is no mean shear across the interface. These experiments are usually conducted by stirring the layers with two horizontal grids of bars which are oscillated vertically with a small amplitude about their mean position (usually in the centre of each layer). This type of experiment has recently been reviewed by Turner (1973).

For the moment we shall concentrate only on one aspect of these experiments. One of the first things to be noticed (Cromwell, 1960) was that the stirring tended to produce sharp density interfaces; a result contrary to one's initial expectation. These experiments are examples of what Turner (1973, p. 116) calls "external mixing processes"; i.e. the energy for the mixing is supplied from a region (the centre of the layers) external to the region where the mixing occurs (the interface). We noted earlier that the observation that an interface became thinner with time implied that energy was being supplied in addition to that which is available locally in the mean shear. Hence, the question arises: do external mixing processes always lead to sharp interfaces?

In order to answer this question, consider the case of two layers separated by a density interface of thickness δ, stirred continuously. Fluid is transported across the interface and so the density difference $\Delta\rho$ decreases with time. The centre of mass of the system is being raised by the mixing, and so the potential energy of the density stratification increases with time. From (2.2) we get that the time rate of change of potential energy is given by

$$\frac{\partial(\text{P.E.})}{\partial t}=\frac{1}{6}\left(2g\Delta\rho\delta\frac{\partial\delta}{\partial t}+g\delta^2\frac{\partial(\Delta\rho)}{\partial t}-3gD^2\frac{\partial(\Delta\rho)}{\partial t}\right).$$

Rearranging this equation we obtain an expression for the change of interface thickness with time: viz,

$$\frac{\partial\delta}{\partial t}=\frac{1}{2g\Delta\rho\delta}\left(6\cdot\frac{\partial(\text{P.E.})}{\partial t}+g(3D^2-\delta^2)\frac{\partial\Delta\rho}{\partial t}\right). \tag{3.3}$$

Now $\partial(\text{P.E.})/\partial t>0$ as explained above, whilst $\partial\Delta\rho/\partial t<0$, and so the sign of $\partial\delta/\partial t$ depends on which term on the right hand side of (3.3) has the larger magnitude.

Situations have certainly been found when $\partial\delta/\partial t<0$ (see e.g. Figure 9, Crapper and Linden, 1974), and there appears to be a wide parameter range when $\partial\delta/\partial t=0$ and the interface thickness remains constant. The case where the interface becomes thicker with time ($\partial\delta/\partial t>0$) is documented here for the first time. It can readily be shown, using values of entrainment rates taken from Turner's (1968) experiments, that $\partial\delta/\partial t>0$ should occur as $Ri_0\rightarrow 0$. The Richardson number is defined in terms of the velocity, u, and length scale, l, of the grid-generated turbulence at the interface in the form $Ri_0=g\Delta\rho l/\rho u^2$. This limit can be achieved by letting $\Delta\rho\rightarrow 0$ in these experiments. Results of three density profiles taken through an interface at different Ri_0 are shown on Figure 3. In Figure 3(a) ($Ri_0=9.4$) a well defined interface is observed with two relatively uniform layers on either side. At a smaller value of Ri_0 the interface begins to become less well defined (Fig. 3(b), $Ri_0=5.8$), and for $Ri_0=3.8$ (Fig. 3(c)) the transition region is extremely thick. Thus, we can conclude, that an external mixing process does not always lead to sharp interfaces. This is really to be expected, with hindsight, as it is in these final stages as $\Delta\rho\rightarrow 0$ that the two layers mix completely.

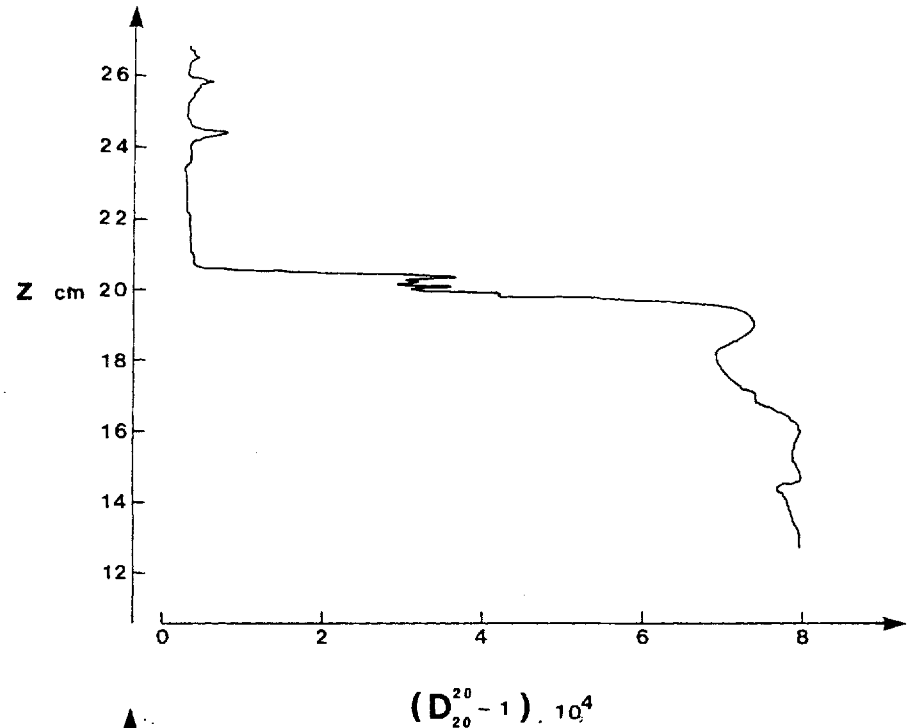
This result has the important consequence that it implies that a change in the structure of the density interface does not necessarily mean that a different kind of mixing process is active.

4. ENERGY PARTITION

We have seen, from the discussion of Koop's (1976) experiments, that energy arguments ignoring dissipation, whilst possibly a qualitative indication of what is going on, can be quantitatively in serious error. It is, therefore, important to know how the energy is partitioned in a stratified, turbulent flow.

In the unstratified case, energy put into the motion at some scale cascades to higher and higher wavenumbers until it is dissipated to heat by viscosity. When a small stratification is imposed, it seems that some of this energy is removed during the cascade to do work against the buoyancy forces. This work can either appear as potential energy produced by mixing, or can generate internal waves which are eventually dissipated by viscosity without contributing to mixing. The fraction of the available kinetic energy which appears as the potential energy of the change in the stratification is called the flux Richardson number Rf. As Ri_0 increases from zero, so too does Rf. As $Rf<1$, the question of what happens to Rf as Ri_0 increases becomes relevant. Rf is a measure of the efficiency of the mixing.

One obvious kind of behaviour is that Rf increases to some value, Rf_c say, and then remains constant thereafter. This was suggested by some early theoretical work of Ellison (1957) and Townsend (1958) and is very appealing from a physical point of view. It implies that the buoyancy forces extract a maximum fraction of the available energy and further



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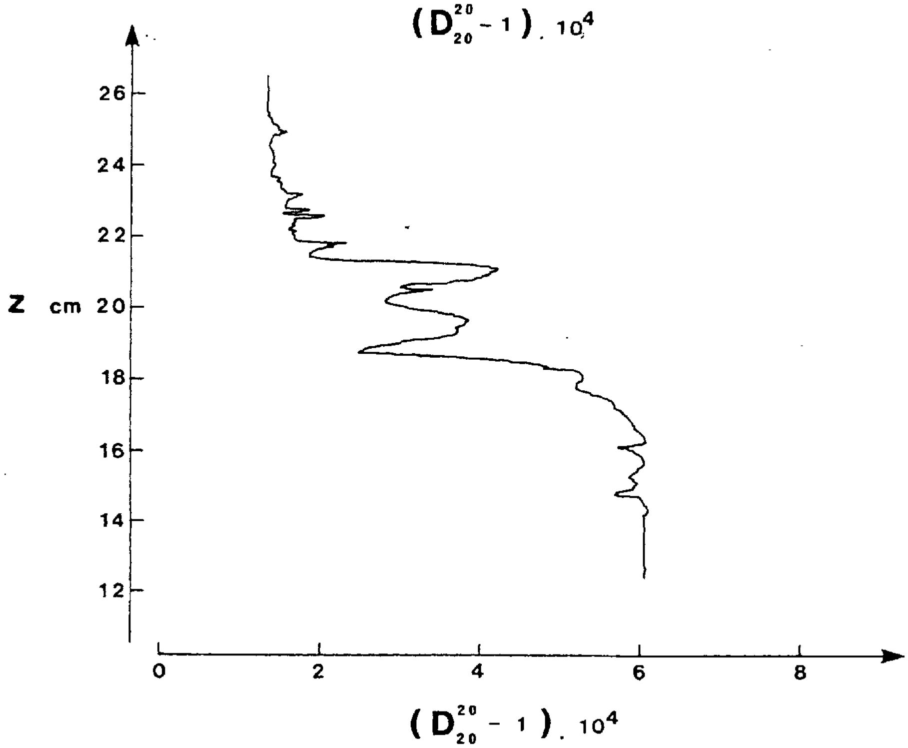

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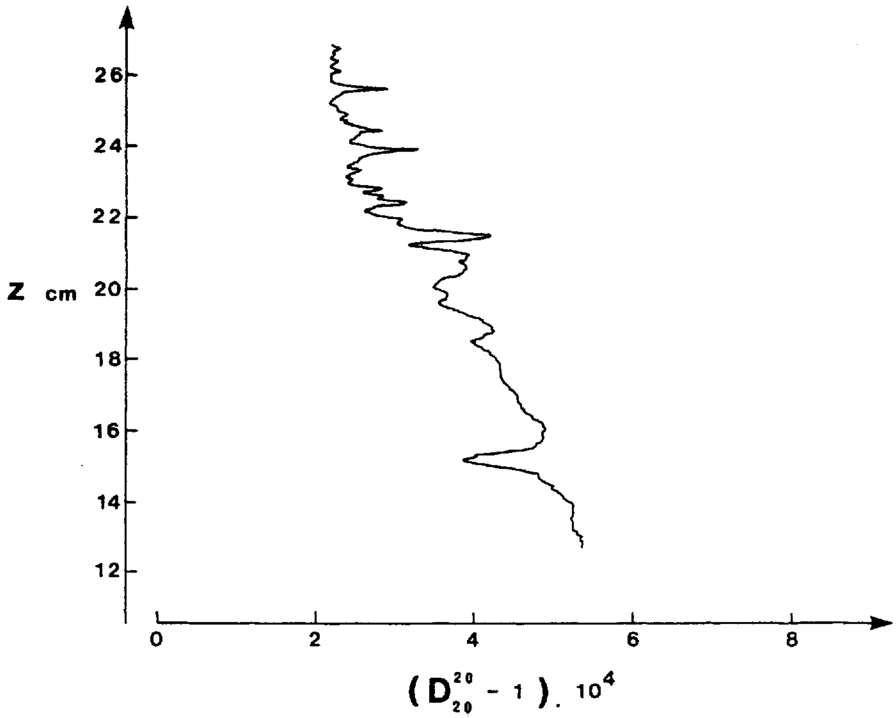

FIGURE 3 Plots of density versus depth for three values of the Richardson number (a) $Ri_0=9.4$, (b) $Ri_0=5.8$, (c) $Ri_0=3.8$. The profiles were measured by driving a conductivity probe vertically through the interface between two layers stirred by horizontal grids. The grids are placed at 10 cm and 30 cm above the bottom of the tank($z=0$) and the interface was initially at $z=20$ cm. The density difference between the layers is due to dissolved salt. Ri_0 is defined in terms of this density difference and typical velocity and length scales of the turbulence near the interface in the manner of Turner (1973, p. 290).

increase in stratification does not affect this value. However, as we shall see, this does not appear to be the case.

Direct measurements of Rf as a function of Ri_0 have been made in a number of experiments by determining the change in the density stratification due to the mixing. A summary of the results of these investigations is shown on Figure 4.

The experiments of Koop (1976) have already been discussed. Thorpe (1973) examines the mixing produced by Kelvin–Helmholtz instability at the interface between two, counter-flowing layers. Both of these authors quote very similar values of Rf, with Rf decreasing as Ri_0 increases. However, as $Ri_0 \rightarrow 0$, $Rf \rightarrow 0$ and so, although they produce no data in that

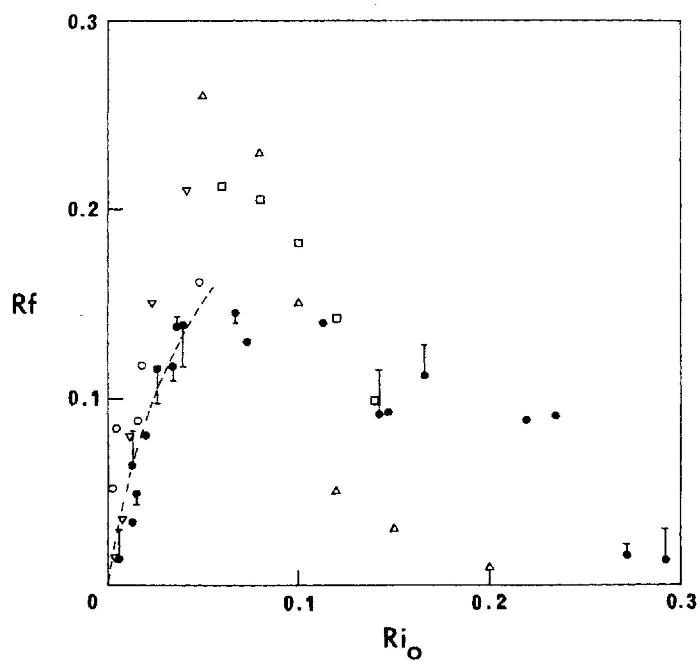

FIGURE 4 The flux Richardson number *Rf* plotted against the overall Richardson number Ri_0 for a number of different experiments. ●, data from mixing produced by dropping a grid of square bars through a density interface. ▽, mixing induced by firing a number of vortex rings at an interface. ○, values calculated from density profiles measured in the wake of a vertical plate by Prych, Harty and Kennedy (1964). □, △ are values of *Rf* measured by Thorpe (1973) and Koop (1976), respectively, for mixing induced by shear instability at an interface. The broken line is an approximate representation of the data of Grigg and Stewart (1963). In the last three cases Ri_0 is taken to be the value given by the respective authors without adjustment. In the other three experiments Ri_0 is defined in the text.

range, the curve of *Rf* must rise to a maximum and decrease again with increasing Ri_0.

Prych, Harty and Kennedy (1964) studied the wake produced at an interface when a plate of height b is moved horizontally along it with a velocity V normal to its face. They present density profiles downstream of the plate and it is possible to calculate the fraction of energy used in mixing. We define Ri_0 for this configuration to be $Ri_0 = g\Delta\rho b/\rho V^2$ and find that Rf=0.05, 0.08, 0.09, 0.12, 0.16 for Ri_0=0.005, 0.008, 0.032, 0.036, 0.092 respectively. Thus *Rf* increases with stability in this case (see Figure 4).

Grigg and Stewart (1963) studied the mixing produced by small turbulent blobs injected vertically into a constant density gradient. They found that Rf increased with stability. We have sketched on Figure 4 the range of their experimental results: these data are taken from Figure 7 of their original paper.

The results of two studies by the author are also presented on Figure 4. The first concerns the mixing produced when a series of vortex rings were fired at the interface (Linden, 1970). In this instance $Ri_0 = (g\Delta\rho l/\rho u^2)\cdot(\pi l^2/4A)$ where u is the velocity of propagation of the ring, l its diameter and A the surface area of the tank. Thus Ri_0 is an average over the whole interface. As indicated on Figure 4, Rf increases with Ri_0.

The second study examines the mixing produced by a horizontal grid of bars which falls freely under gravity through a density interface. The amount of energy input is determined simply by the change in potential energy of the grid† (its final kinetic energy is negligible), and the fraction, Rf, of this used in mixing is determined by measuring the stratification. The overall Richardson number is taken to be $Ri_0 = g\Delta\rho b^2/\rho U^2 M$, where b is the width of the grid bars, M the mesh length and U the terminal velocity in pure water. The results on Figure 4 show Rf increasing to a maximum at $Ri_0 \simeq 0.1$ and then decreasing with further increase in Ri_0. So far only one grid geometry has been considered (1 cm square bars with 5 cm between centres). Typical Reynolds numbers $Re = Ub/\nu$ are $0(10^3)$.

The encouraging feature about Figure 4 is that the results from a number of different experiments fit together quite well. The general picture is that Rf increases at small Ri_0 until it attains a maximum and then decreases again at high Ri_0. There is some discrepancy between the maxima from the different experiments, although at most something between 1/6–1/4 of the total energy appears to be converted to potential energy. The shear induced mixing processes (Koop, 1976; Thorpe, 1973) appear to be the most efficient. Only one set of experiments (the falling grid) give the full range of behaviour and it would be interesting to see what happened if the parameter ranges of the other experiments were extended.

It seems likely that at high values of Ri_0 a significant fraction of the turbulent kinetic energy is used to generate internal waves. Provided these waves do not break they do not contribute to mixing, but are eventually dissipated by viscosity. Whether or not the drop in Rf from its maximum value at high Ri_0 is due to the generation of internal waves is not yet known, but there are some indications that this might be the case. If $t_E = l/u$ represents the turnover time of an eddy and $t_B = (g\Delta\rho/\rho l)^{-1/2}$ the relaxation time of the buoyancy field to a disturbance of scale l, then Ri

†This value depends on the height the grid falls, which was kept constant in these experiments. Although the magnitude of Rf depends on the height used, the *shape* of the Rf versus Ri_0 curve is unaffected.

$= g\Delta\rho l/\rho u^2 = (t_E/t_B)^2$. From considerations of the mixing produced by a vortex ring at an interface, Linden (1973) finds that, the response of the interface scales with t_B. Thus, the mixing is limited by a wavelike response of the interface, which inhibits the conversion of turbulent kinetic energy into potential energy of the stratification.

Before concluding this section there are two other points to be made concerning the flux Richardson number. These are both related to the fact that the change in potential energy of the stratification is due to the raising of the centre of mass of the fluid. This is completely equivalent to the entrainment of fluid across a density interface. In the grid mixing experiments referred to in Section 3, the rate of increase of potential energy of the stratification is $\frac{1}{2}g\Delta\rho u_e D^2$, where u_e is the entrainment velocity. The rate of supply of kinetic energy is $\frac{1}{2}\rho u^3 D$. Thus

$$Rf = \frac{u_e}{u}\frac{g\Delta\rho D}{\rho u^2} \propto \frac{u_e}{u} Ri_0.$$

In the case where molecular diffusion is unimportant, it is found experimentally that

$$\frac{u_e}{u} \propto Ri_0^{-n},$$

and so

$$Rf \propto Ri_0^{\,1-n}. \tag{4.1}$$

This implies that Rf is an increasing function of Ri_0 for $n<1$, and a decreasing function for $Ri_0>1$. In the case $n=1$, a constant fraction of the kinetic energy supply is used for mixing; this simple form has often been assumed to be the case when trying to predict the entrainment velocity. There seems to be little justification for this assumption. In fact, Turner (1968) and Hopfinger and Toly (1976) find that at high Peclet number for $Ri_0 \gtrsim 7$, $n=3/2$, which implies that Rf is a decreasing function of Ri_0. On the other hand for $Ri_0 \lesssim 7$, the exponent n is smaller, and although there are few data points it seems certain that $n<1$ for small enough Ri_0. Thus we have very similar qualitative behaviour to that shown on Figure 4, in the grid mixing experiments also. It is worth noting here that mixing driven by the application of a surface stress on a region of stratified fluid also produces this type of behaviour (Kantha, Phillips and Azad, 1977).

Finally, we note that $u_e\Delta\rho$ is the flux of density across the interface. If we define an eddy diffusivity, K, as this flux divided by the mean gradient

$(\Delta\rho/D)$ we readily find that

$$\frac{K}{\kappa} \propto Rf\, Ri_0^{-1} Pe \tag{4.2}$$

where $Pe = UD/\kappa$ is the Peclet number. The concept of eddy diffusivity is only likely to be useful in the limit $Ri_0 \to 0$, when the buoyancy forces are very weak. Thus the slope of the Rf vs. Ri_0 curve at the origin on Figure 4 is proportional to the eddy diffusivity.

5. FORMATION OF INTERFACES

Up to this point we have considered mixing at existing density interfaces. Now we turn our attention to the processes by which an interface can form in a uniformly stratified region.

Recently, an attractive proportion has been put forward by Posmentier (1977). He suggests, and this is an extension of an idea formulated by Phillips (1972), that if the vertical turbulent flux of density decreases as the vertical density gradient increases, then any perturbation causing an increase in the gradient will be amplified. This is because the density flux will be smaller at this point than at points above or below it where the gradient is smaller. Consequently, mass will accumulate at this point increasing the density gradient still further. This is in contrast to the more familiar situation where the flux of density increases with increasing density gradient, as it does if there is a constant eddy diffusivity. Then mass will be transported away from the region of highest gradient and so any irregularity will tend to be smoothed out.

In a given turbulent situation, if we plot the vertical density flux F as a function of the density gradient $d\rho/dz$ we might expect the situation shown on Figure 5: as $d\rho/dz$ increases from zero, F increases from zero: at large

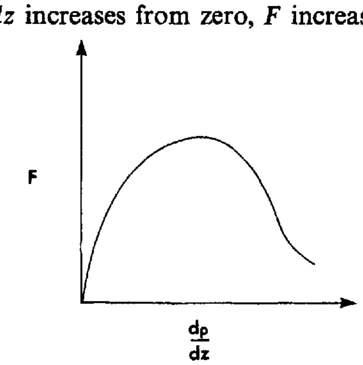

FIGURE 5 A diagrammatic representation of the density flux F as a function of the density gradient $d\rho/dz$.

values of $d\rho/dz$ the high stability might lead to a diminution of the flux. If it is of this form we would get a different behaviour on either side of the maximum of F. To the left, interfaces would tend to be smeared out: to the right they would be intensified.

The difficulty with this approach lies in determining the curve of F against $d\rho/dz$ in a given situation. It also requires that the flux *only* depends on the density gradient. This requires, as pointed out by Puttock (1976), who had independently proposed the same mechanism, that the scales of the turbulence must be small compared with the regions of density gradient.

In the case of the shear-induced mixing (Thorpe, 1973; Koop, 1976) the scales of the turbulence are comparable to, or larger than, the density gradient regions and so we cannot apply these arguments. However, the grid mixing experiments do seem to satisfy our criteria and we will consider them in some detail below.

Consider the case where turbulence exists throughout the fluid. In Section 4 we noted that the rate of change of the potential energy of the stratification is proportional to the buoyancy flux. Consequently, for a fixed rate of working the flux Richardson number is proportional to the buoyancy flux and the mean density gradient is proportional to Ri_0, and so we can plot Rf as a function of Ri_0. But this is precisely what we have in Figure 4, and as noted earlier, $Rf(Ri_0)$ has the required shape.

This behaviour was also found in the grid mixing experiments as was mentioned above. We also showed above that an entrainment rate of the form $u_e/u \propto Ri_0{}^{-n}$ implies that Rf is an increasing function of Ri_0 for $n<1$ and decreases if $n>1$ (see (4.1)). From Turner's original data (see Turner 1973, Fig 9.3) we see that for a salinity interface $n>1$ for $Ri_0 \gtrsim 7$, but that n decreases at smaller Ri_0. On the basis of Posmentier's argument, we should expect a sharp interface to be maintained for $Ri_0 \gtrsim 7$, but for it to be smeared out at lower values of the overall Richardson number. This conclusion is in qualitative agreement with that obtained from the energy arguments presented in Section 3. Furthermore, it is in excellent quantitative agreement with the experimental results shown on Figure 3.

In the experiments where a grid is dropped through an initially sharp interface the situation is not so clear. The behaviour of Rf versus Ri_0 has the correct form to provide both intensification and smearing out of interfaces for different values of Ri_0. However, as the experiments begin with an extremely sharp interface it is difficult to see how intensification can take place. Indeed, a similar difficulty underlies the grid mixing experiments. When the stirring is begun on both sides of a very sharp interface it will initially be smeared until its thickness is approximately equal to the integral scale of the turbulence (Crapper and Linden, 1974).

The reason that the interface in the grid stirred experiments has this thickness at large Ri_0 (>5) is not known. It is possibly related to the fact, mentioned above, that for the arguments based on density flux to apply we require that the scales of the turbulence be small compared with the region of density gradient. In the grid mixing experiments this is only beginning to be achieved when the interface thickness becomes comparable with the integral scale of the turbulence. This is also the reason why it is inappropriate to use the grid dropping experiments to directly test these ideas. A more relevant experiment is to drop a grid through a linearly stratified region: these experiments are in progress.

6. SOME FINAL REMARKS

In this paper I have concentrated on some of the aspects of mixing in a stably stratified fluid which have been the subject of laboratory experiments. Possibly the most interesting conclusion that we can draw from this discussion is the fact that it seems to be possible for turbulence to exist no matter how high the overall Richardson number is. The flow adjusts so that there are regions in which there are no significant velocity gradients but only weak density gradients. In this way small gradient Richardson numbers are obtained which allow instabilities to develop. However, the turbulence so produced is localised in space and time and has the intermittent character we have come to expect in these stable conditions.

One major omission in this discussion has been that of the measurements of entrainment in stratified flows, but these have been reviewed by Turner (1973) and, more recently, by Sherman, Imberger and Corcos (1978). One of the objects of the entrainment measurements is to use the results in geophysical situations such as in modelling the oceanic mixed layer. This raises the questions of Reynolds number effects which as Turner (1973) points out, may be particularly severe in a stratified fluid due to the restriction in length scales imposed by the stratification.

There is very little information about the effects of Reynolds number, or of other molecular properties of the fluid. Koop (1976) has investigated the effect of change in Reynolds number for his shear flow experiments. He found that, although the initial instability is independent of *Re*, the final thickness of the density interface does seem to be larger at higher *Re*. He attributes this to the settling out of unmixed fluid at lower values of the Reynolds number. Crapper and Linden (1974) found that the structure of a density interface between two stirred layers had a strong dependence on Peclet number at low values of *Pe*. Turner (1968) had previously noted that the entrainment rate across an interface was different at different Peclet numbers.

Precisely what these molecular effects are is an important and unresolved question. In the experiments where a grid is dropped through an interface we could imagine the case where the fluids above and below the interface are immiscible. This corresponds to infinite Peclet number, and clearly the final state must be the same as before the grid was dropped, and so $Rf=0$, for all Ri_0. Presumably energy is stored as surface tension energy of droplets, but provided emulsification does not occur, this will be released as the drops recombine into their original layers. It seems, therefore, that these molecular effects may be extremely important in determining the ultimate efficiency of a mixing process. There is a need, therefore, to design experiments in which these phenomena can be isolated and examined. The grid-dropping configuration appears to be a suitable one and experiments aimed at investigating some of these effects are in progress.

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