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A note on the transport across a diffusive interface

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Abstract—The ratio of the fluxes of heat and salt across a diffusive interface is investigated on the premise that there are two contributions to the flux of each component. These contributions arise from the two transport mechanisms which can operate at a diffusive interface, viz. double-diffusive convection and turbulent entrainment. Estimates of the fluxes are provided for each process, and their combined effects are examined and compared with the experimental results of TURNER (1965) made in a heat–salt system. It is found that, by simply adding these two contributions, it is possible to explain the observed changeover from the constant regime to the variable regime which is in satisfactory agreement with the data. An estimate is also given for a sugar–salt system which is consistent with the, as yet semi-quantitative, observations. It is shown that turbulent mixing is only important in the variable regime, and in these circumstances, large fluxes of a component will be observed.

THE DISCOVERY of temperature and salinity microstructure indicating the presence of interfaces separating a series of convecting layers (e.g. TAIT and HOWE, 1968, DEGENS and ROSS, 1969) has led to an interest in the coupled vertical transports of heat and salt in such circumstances. As a result, the fluxes of heat and salt have been measured across both 'diffusive' and 'finger' interfaces (TURNER, 1965, 1967; CRAPPER, 1973; LINDEN, 1971, 1973a) formed between two layers by double-diffusive convection (see TURNER, 1973, Ch. 8). Of particular interest here are the laboratory measurements of the fluxes made across a diffusive interface, which are described briefly below.

TURNER (1965) and CRAPPER (1973)† have measured the fluxes of heat F_T and salt F_S across a diffusive interface, and related their data to the difference in temperature ΔT and salinity ΔS between the layers. In case of a diffusive interface, the lower layer is both warmer and saltier than the upper layer but is more dense: i.e. $R\rho = \beta\Delta S/\alpha\Delta T \geq 1$, where α and β are the proportional density changes for a unit change in temperature and salinity, respectively, and Δ denotes the magnitude of the change across the interface. They found that the measured ratio of the fluxes $R_f = \beta F_S/\alpha F_T$ was related to $R\rho$ by a relationship of the form

$$R_f = f(R\rho). \qquad (1)$$

The function f was found to be a constant at a value of approximately 0·15 for $2 \leq R\rho \leq 8$ and $f \rightarrow 1$ as $R\rho \rightarrow 1$. Turner's data and the approximate form of f (shown as the solid line) are given in Fig. 1. The region of constant f has been called the constant regime, and the region of variable f the variable regime (HUPPERT, 1972).

Recently, SHIRTCLIFFE (1973) measured the corresponding flux ratio across a diffusive interface in a sugar–salt system. He found that R_f was approximately constant

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†CRAPPER (1973) has checked TURNER's (1965) experimental results using more accurate measurement techniques; although his results differ from those of Turner in detail, the essential features of and the conclusions drawn from Turner's results are confirmed.

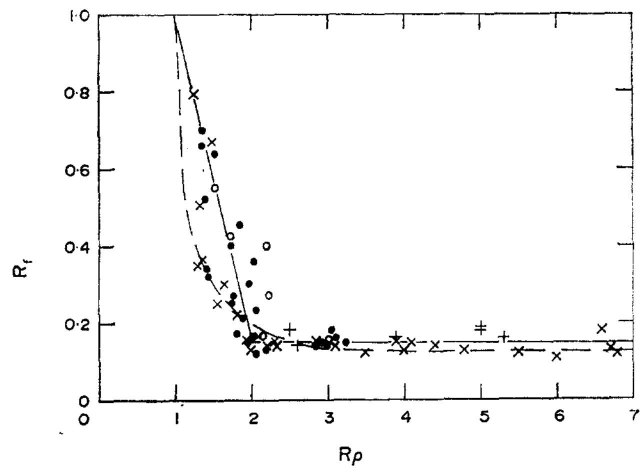

Fig. 1. The ratio of the buoyancy fluxes of heat and salt plotted as a function of the interface stability (taken from TURNER, 1965). The broken curve is a plot of (7).

at a value of 0·59 for all values of $R\rho$ (which included some values very close to unity). Both Shirtcliffe's and Turner's values for R_f in the constant regime are consistent with the notion that

$$R_f = \tau^{\frac{1}{2}}, \tag{2}$$

a result derived by VERONIS (1968) for convection in the absence of an interface in the limit as $R\rho \rightarrow 1$. Here $\tau = \kappa_S/\kappa_T$ where $\kappa_{S(T)}$ is the coefficient of molecular diffusion of $S(T)$. A simple mechanistic argument to explain (2) was given by Veronis in terms of convective instability from the edge of a diffusive interface.

HUPPERT (1972) has studied the stability of a series of convecting layers separated by diffusive interfaces. He showed that when $R\rho$ is in the constant regime, the layers are neutrally stable. However, when the interface is in the variable regime, the interface is unstable and the layers will merge. Also, it has been observed in oceanographic situations that $R\rho$ is close to unity across interfaces where there are opposing mean vertical gradients of temperature and salinity. It is of interest, therefore, to see why the variable regime exists at all and to account for the observed flux ratios.

In the limit $R\rho \rightarrow 1$, the interface will become statically unstable and $R_f \rightarrow 1$ as the interface breaks down. In this note, we investigate the possibility that the variable regime is a result of localized breakdown of the interface (i.e. entrainment), produced by the convective motions acting on it. The object here is to see if the eddies produced by the convection have sufficient energy to entrain fluid across the interface. In accordance with the above ideas, it is assumed that the flux across the interface consists of two parts:

(i) a 'diffusive flux' resulting directly from the double-diffusive instability and drawing its energy directly from the potential energy in the heat stratification. In accordance with the work of VERONIS (1968), we suppose that $R_f^d = \tau^{\frac{1}{2}}$ where the superscript ^{d} denotes the diffusive flux.

(ii) An 'entrainment flux' resulting from the mechanical mixing across the interface by the interaction of the convective motions in the layers (driven by the unstable buoyancy flux across the interface) with the interface.

It is further assumed that these two contributions to the total flux are additive. This assumption requires that the mechanical mixing across the interface does not significantly affect the double-diffusive instability and vice versa. TURNER (1968) has shown that mixing across a density interface due to grid-generated turbulence occurs as an ensemble of relatively rare mixing events, and so this assumption seems justified. If we write the rate at which fluid is mechanically entrained across the interface as an entrainment velocity u_e (defined as the amount of fluid entrained/unit area/unit time) the ratio of the total fluxes of S and T is given by

$$R_f = \frac{u_e \beta \Delta S + \beta F_S{}^d}{u_e \alpha \Delta T + \alpha F_T{}^d},$$

$$= \frac{R\rho\, u_e/u + \tau^{\frac{1}{2}} \alpha F_T{}^d/u\alpha\Delta T}{u_e/u + \alpha F_T{}^d/u\alpha\Delta T}, \tag{3}$$

where u is a typical velocity of the convective motions in the layers.

It is necessary now to obtain an expression for the non-dimensional entrainment rate u_e/u. LINDEN (1973b) has shown that entrainment across a sharp density interface by turbulence with velocity scale u and length scale l at high Reynolds and Peclet numbers takes the form

$$\frac{u_e}{u} = \gamma\, F_r{}^3, \tag{4}$$

where F_r is an interfacial Froude number defined by

$$F_r = u/(gl\Delta\rho/\rho)^{\frac{1}{2}}, \tag{5}$$

and γ is a constant (for a particular geometry). Experimental data [consistent with (4)] obtained by TURNER (1968) indicate that $\gamma \approx 0{\cdot}6$. The relationship (4) is considered appropriate for turbulent entrainment across a diffusive interface, as the interface was observed to remain sharp during the experiments (CRAPPER, 1973). Further, CRAPPER (1973) has measured the r.m.s. temperature fluctuations θ in the convecting layers on either side of a diffusive interface and found that $\theta \sim 0{\cdot}1\ \Delta T$. Noting that

$$F_{rT} = u/(g\alpha\ \Delta T\ l)^{\frac{1}{2}} = (R\rho - 1)^{\frac{1}{2}}\, F_r,$$

and using the estimate of the diffusive flux $\sim u\theta$, we find

$$R_f = \frac{\gamma R\rho\ F_{rT}{}^3 + 0{\cdot}1\ (R\rho - 1)^{3/2}\ \tau^{\frac{1}{2}}}{\gamma F_{rT}{}^3 + 0{\cdot}1\ (R\rho - 1)^{3/2}}. \tag{6}$$

For the heat–salt case, a typical value of F_{rT} (using measurements of the convective velocities in the layers given by CRAPPER, 1973) is about 0·2. Then, with the appropriate values for the molecular coefficients, this gives

$$R_f \approx \frac{(R\rho - 1)^{3/2} + 0{\cdot}5\ R\rho}{10\ (R\rho - 1)^{3/2} + 0{\cdot}5}. \tag{7}$$

Equation (7) is plotted without further adjustment as the broken curve on Fig. 1: the agreement with TURNER's (1965) experimental data is satisfactory. For the case of transport across a diffusive interface in a sugar–salt system the convective velocities involved are much smaller and the values of $\alpha\Delta T$ an order of magnitude larger, and so R_f would remain approximately constant for smaller values of $R\rho$. An estimate of F_{rT} based on semi-quantitative observations in a sugar–salt system indicate that a reasonable value would be approximately 0·05. Then (6) indicates that the change-over from the constant regime to the variable regime would occur at $R\rho \approx 1{\cdot}02$. TURNER and CHEN (in press) estimate, on the basis of the observed stability of a series of layers, that the value of transition for sugar–salt occurs in the range $1{\cdot}06 \leq R\rho \leq 1{\cdot}10$.

A note of caution is appropriate when interpreting (6). For given molecular properties, it was noted earlier that the data were consistent with $R_f = f(R\rho)$. However, (6) shows that the magnitude of the density step across the interface is also an important parameter, in that it affects the efficiency of the mechanical entrainment across the interface. TURNER'S (1965) experiments were not sufficiently accurate to show the variation of R_f with $\Delta\rho$, although the results of CRAPPER (1973) show evidence of this consistent with (6). This is an important point to consider when attempting to apply TURNER'S (1965) data to an oceanic situation. In the oceanic case, the density step across an interface is typically two orders of magnitude smaller than those in Turner's experiments and so the effects of turbulent entrainment may be significantly greater in the former case. The introduction of a Froude number in (6) does also *not* imply the result is valid in that form when motions are externally imposed on the interface. This is because the velocities considered in (3) are convectively driven and an extra parameter is required which relates the intensity of the convectively driven motions to that of the externally imposed motions.

One further feature of transport across a density interface can be discussed on the basis of the model proposed above. TURNER (1965) showed that the magnitude of the flux of heat across the interface was greater than the 'solid-plane value', which is the value appropriate to molecular diffusion across a rigid conducting plane placed at the interface. This increase in the flux of heat was explained by Turner as due to the increase in surface area produced by contorting the interface: HUPPERT (1972) interpreted this increase as due to the weaker constraints imposed on the horizontal velocity fields by the interface. The larger flux can be interpreted on the basis of this model as produced by the mechanical entrainment across the interface. Writing, as before, the total heat flux as the sum of the diffusive flux and entrainment flux, it is easy to show that in the limit as $R\rho \to 1$, the total heat flux/'solid-plane' value

$$\to \gamma F_{r_T}{}^3\, u(\alpha\Delta T)^{-1/3}\,(R\rho - 1)^{-3/2}.$$

This result expresses the fact that in the limit as $R\rho \to 1$ the interface becomes unstable and entrainment causes an increasingly large flux.

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